# Statistics:
## Bayes' Theorem

Bayes' Theorem (or Bayes' Rule) is a very famous theorem in statistics. It was originally stated by the Reverend Thomas Bayes.

If we have two events A and B, and we are given the conditional probability of A given B, denoted P(A|B), we can use Bayes' Theorem to find P(B|A), the conditional probability of B given A.

**Bayes' Theorem:**
$$\boxed{\mathrm{P(B|A)} = \frac{\mathrm{P(A|B)P(B)}}{\mathrm{P(A|B)P(B)} + \mathrm{P(A|B')P(B')}}}$$

where P(B′) is the probability of B not occurring.

## Example:

**Q:** In a factory there are two machines manufacturing bolts. The first machine manufactures **75%** of the bolts and the second machine manufactures the remaining **25%**. From the first machine **5%** of the bolts are defective and from the second machine **8%** of the bolts are defective. A bolt is selected at random, what is the probability the bolt came from the first machine, given that it is defective?

**A:**
Let A be the event that a bolt is defective and let B be the event that a bolt came from Machine 1.

**Check that you can see where these probabilites come from!**

$$\mathrm{P(B)} = 0.75 \quad \mathrm{P(B')} = 0.25 \quad \mathrm{P(A|B)} = 0.05 \quad \mathrm{P(A|B')} = 0.08$$

Now, use Bayes' Theorem to find the required probability:

$$\mathrm{P(B|A)} = \frac{\mathrm{P(A|B)P(B)}}{\mathrm{P(A|B)P(B)} + \mathrm{P(A|B')P(B')}}$$

$$= \frac{0.05 \times 0.75}{0.05 \times 0.75 + 0.08 \times 0.25}$$

$$= 0.3846$$

## Try this:

**Exercise:** Among a group of male pensioners, 10% are smokers and 90% are nonsmokers. The probability of a smoker dying in the next year is 0.05 while the probability for a nonsmoker is 0.005. Given one of these pensioners dies in the next year, what is the probability that he is a smoker?